State Curriculum Framework-2011

Position Paper

on

Mathematics

School Education Department
Telangana, Hyderabad.

State Council of Educational Research & Training,
Telangana, Hyderabad.

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Vision of the State

The vision of the State is that ALL children should receive high quality education and become responsible citizens with an acute sense of the other. They should be aware of their environment and think about it critically. They should listen carefully and speak fearlessly. They should be able to understand what they hear and read; but they should also be able to questions it. Teachers should promote these skills, provide meaningful teaching learning processes in natural and friendly environment that enable children to express themselves freely and ask questions. Teachers are collaborative learners and reflective practitioners. Parents and community should have a sense of ownership and participate in the life of the school. In a world which is becoming increasingly instrumental, materialistic and competitive, school should become a space for reflection, cooperation and promotion of human and ethical values.

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EXECUTIVE SUMMARY

Mathematics is the result of the human mind's ability to abstract from life. It consists of ideas and concepts like numbers which though created for counting have no relationship to the individual characteristics of the objects being counted. Mathematics uses both deductive and inductive reasoning to build itself. What are these? When statements or propositions are based on a set of observations and experiences, drawing on patterns observed and generalizing from them, it is called inductive reasoning. On the other hand when truth is established through the process of deductive reasoningit is based on a previously established statement and logic. Such previously established statements are statements which are self-evident truths i.e statements we assume to be true as they seem intuitively correct (axioms and postulates) or statements that are assumed as the basis of the mathematics objects constructed or are ones which have been proven true based on the self evident truths(i.e theorems).

For some the value of Mathematics lies in its practical utility i.e. in its application in day-to-day life and work. Others appreciate it as a tool for improving thinking and for some others it is simply pursued because they enjoy it. All three of them are valid reasons for engaging with the discipline and have implications for teaching mathematics in school. By including mathematics in the school subjects we want children to develop skill and understanding in the various curricular areas related to number and space and logical thinking. They should develop problem solving abilities- understand/formulate problems, develop a variety of strategies to solve them, verify and interpret results, generalize to new situations.

Children have many innate abilities for Mathematics such as those of classification, matching, estimation, analysis, mapping and generalization. The conceptions of number and space- more-less, far-near, big-small, in-out, tall-short, heavy-light etc. should be utilized in introducing them to formal Mathematics.

Children seek to make sense of school Mathematics in their lives. Thus, any attempt to develop teaching learning material for children or engage with them in the classroom must respect their everyday life experiences. In many poor urban households, children participate in economic activities. Almost no school curriculum gives any place to such

everyday ‘street Mathematics’. Beginning from these concrete experiences children can be helped to move to more abstract mathematics.

Mathematics suffers from the syndrome of one correct answer. Children need to be encouraged to come up with and use more than one way/method of doing things and solving problems.

The Mathematics syllabus for the elementary classes has to revolve around understanding and using numbers and the system of numbers, understanding shapes and spatial relations, measurement, handling data etc. In this the identification of patterns is central as it helps children make the transition from arithmetic to algebra. But as they move to secondary classes the logical structure of mathematics should become more evident to them. Geometry and trigonometry, number Theory; e.g., tests of divisibility; properties of the sequence of positive integers, algebra are some areas wherein students can learn to appreciate this structure best.

Managing a Mathematics classroom requires a teacher to engage all children in the classroom. This is possible when a teacher understands that all children can learn Mathematics. Various prevelant social discriminations of caste and gender also work in the context of Mathematics education. Children with special needs especially those with mental and physical disabilities, have to be given equal importance as other children, with due care towards their disability. They need to be respected as human beings.

Like other disciplines, Mathematics education also needs preparation with regard to understanding the nature of the subject, its content and pedagogic techniques of Mathematics. Mastery of the subject is an absolute essential for effective teaching; the level of teacher knowledge and understanding should be much higher than that he is expected to impart. A teacher’s personal characteristics are also important. These include general appearance, health, energy, enthusiasm, co-operation, sense of humor, sense of justice and willingness to speak the facts. Each teacher should make definite provisions for his own professional growth and development. Institutional support is important in this context — support from the State educational institutions, and also from the authorities in the school to which the teacher belongs. This includes membership of professional bodies, and taking part in teacher development programmes which they conduct from time to time. Every teacher must develop a network of co-teachers and teachers of higher level class teachers for clarification of doubts. Teachers should give students ample space in the classroom so that they can discuss, debate, voice their opinion and critique.

Teacher must consciously undertake continuous evaluation of the student for better teaching and comprehensive assessment of the student. It is important that assessment should be based on more than just computational ability and committing to memory many formulas and solved problems; an assessment system must also test for conceptual understanding.

1. THE NATURE OF MATHEMATICS

Mathematics as an expression of the human mind reflects the active will, the contemplative reason and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality.

What is Mathematics?

Mathematics is the result of the human mind's ability to abstract from life. Created by the mind to count, numbers have no relationship to the individual characteristics of the objects being counted. The number 3 is an abstraction from all the actual collections of three apples, three books, three girls etc. Similarly, the triangle is an abstraction and has no relationship with the shape of the sail or a rooftop.

Mathematics does not limit itself to such abstractions. It uses such abstractions to build many more abstractions, for e.g. even numbers, odd numbers, prime numbers are abstractions, building on the abstraction of numbers. Thus, Mathematics learns from human experiences, but its growth and progression is not dependent on them; what it relies on is, the logic and creativity of the human mind.

--Mathematics studies number and space. It answers many questions like the one listed below-

- How many objects are lying on the table?
- Do you have more than me?
- How much does he weigh? Does he weight more than me?
- How tall is she?
- How much more distance do I have to cover?
- How much area does this object occupy?
- Will this object fit in this space?
- How will this object look if it is viewed from (1) top (2) from the side (3) from the bottom?
- How will the object look if it moves from 'a' position to 'b' position?

--Mathematics also studies patterns and relationships in the domain of numbers and space. Thus, question like the ones given below are also of interest in the discipline-

- Does the interval between primes form a pattern?
- Is there a relationship between the change in surface area of a regular solid and its volume?

Mathematics uses both deductive and inductive reasoning to build itself. When statements or propositions are based on a set of observations and experiences, drawing on patterns observed and generalizing from them, such reasoning is called inductive. The basis of an inductive generalization is thus a collection of empirically verifiable statements.

An example of inductive reasoning in Mathematics-

We may notice that the following numbers are all primes:

- $2^1 + 1 = 3$
- $2^2 + 1 = 5$
- $2^4 + 1 = 17$
- $2^8 + 1 = 257$
- $2^{16} + 1 = 65{,}537$

From this we may inductively conclude that so will be the numbers $2^{32} + 1$, $2^{64} + 1$, and so on. But later we will be forced to modify this hypothesis when we find that the next number in this sequence ($2^{32} + 1 = 4{,}294{,}967{,}297$) is not a prime number. Such a situation did actually happen: the hypothesis was made by Pierre Fermat, and it was disproved by Leonhard Euler.

Or, we may notice that every prime number which is 1 more than a multiple of 4 can be written as a sum of two perfect squares, for example:

- $5 = 2^2 + 1^2$
- $13 = 3^2 + 2^2$
- $17 = 4^2 + 1^2$
- $29 = 5^2 + 2^2$

From this we may make an inductive generalization to the effect that every such prime can be written in this way, and every additional such prime number we test will add to our conviction that this is the case. This hypothesis turns out to be correct; it is called the Fermat Two Squares Theorem. For it to be accepted to be true it has to be proved through well formed logical arguments.

Mathematics establishes the truth of its statements through the process of deductive reasoning. Here we arrive at a statement based on a previously established statement and logic. Such previously established statements are statements which are self-evident truths i.e statements we assume to be true as they seem intuitively correct (axioms and postulates) or statements that are assumed as the basis of the mathematics objects constructed or are ones which have been proven true based on the self evident truths(i.e theorems). Logic used is both inductive and deductive.

An example of deductive reasoning in Mathematics

Is the product of two even numbers always even?

Since the two numbers are even they are divisible by 2. (by definition)

Thus, the numbers can be written as 2m and 2n

The product of 2m and 2n is

2m*2n= 4mn

Since the product is divisible by 2 it is even.

Thus, the product of two even numbers is always even.

Another example

$$1 + 2 = 3 \quad = \quad \frac{2(2+1)}{2}$$

$$1 + 2 + 3 = 6 = \quad \frac{3(3+1)}{2}$$

$$1 + 2 + 3 + 4 = 10 = \frac{4(4+1)}{2}$$

We include that

$$1 + 2 + 3 + 4 + \text{-----} + n = \frac{n(n+1)}{2}$$

How do we prove this:

Assume $1 + 2 \text{ - - -- } + m = \frac{m(m+1)}{2}$

Let us find the sum for m + 1 numbers

$1 + 2 \text{ - - - } + m + m + 1 = (1 + 2 + \text{-------} m) + m + 1$

$$= \frac{m(m+1)}{2} + m + 1$$

$$= \frac{(m+1)(m+2)}{2}$$

Hence we can say

$$1 + 2 + 3 + \text{-----} r \frac{n(n+1)}{2}$$

Actually Mathematics grows in precisely such a manner. These two forms of reasoning need to go hand in hand. We observe a phenomenon, notice a pattern in it and inductively generalize from that. Then we can attempt to establish it deductively (i.e., to prove it), using the available body of knowledge as base. Hence for the subject to advance there must be an opportunity for knowledge obtained to be organized, linked and gradually introduced. This is true at every level of learning of the subject, from school to research.

The distinction between inductive and deductive reasoning also underscores a subtle difference between Science and Mathematics; for in Science most reasoning is inductive. There are no precise equivalent of deductive reasoning in Science except in model building. It is not possible to prove a statement or a model in Science in quite the way it is done in Mathematics. In Science at best one can say: “This statement is consistent and compatible with all known observed phenomena.” Nevertheless, the techniques of Mathematics are essential for progress in Science.

2. OBJECTIVES OF TEACHING MATHEMATICS

For some the value of Mathematics lies its practical utility i.e. in its application in day-to-day life and work. Others appreciate it as a tool for improving thinking and for some others it is simply pursued because they enjoy it. Keeping all this in sight, Mathematics teaching has the following objectives -

1. Children should be able to develop skill and understanding in the various curricular areas related to number and space.
2. Children should be able to reason mathematically.
3. Children should be able to pursue assumptions to their logical conclusion
4. Children should be able to handle abstraction.
5. Children should develop problem solving abilities- understand/formulate problems, develop a variety of strategies to solve them, verify and interpret results, generalize to new situations)
6. Children should develop confidence in using Mathematics meaningfully.

> The narrow aim of school Mathematics is to develop 'useful' capabilities, particularly those related to numeracy- numbers, number operations, measurements, decimals and percentages. The higher aim includes developing the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things, and the ability and the attitude to formulate and solve problems.
>
> NCF,2005

3. NATURE OF THE CHILD – ABILITY OF THE CHILD TO DO MATHEMATICS

1. All children are capable of learning Mathematics.
2. All children are curious by nature and interested in learning. They ask about how things happen and why they happen.
3. Children learn in a variety ways- through experience, making and doing things, experimentation, reading, discussion, asking questions, observation, learning,

playing with puzzles, thinking and reflecting, expressing oneself in speech, movement and writing.

4. Children do not only learn individually but also in their interactions with other children.

5. Children do not come to school as blank slates. Children have many innate abilities for Mathematics such as those of classification, matching, estimation, analysis, mapping and generalization. The conceptions of number and space-more-less, far-near, big-small, in-out, tall-short, heavy-light etc. should be utilized in introducing them to formal Mathematics.

6. Children seek to make sense of school Mathematics in their lives. Thus, any attempt to develop teaching learning material for children or engage with them in the classroom must respect their everyday life experiences.

4. VISION FOR SCHOOL MATHEMATICS

1. Children learn to enjoy Mathematics rather than fear it.
2. Children learn that Mathematics is much more than formulae and mechanical procedures.
3. Children see Mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.
4. Children pose and solve meaningful problems.
5. Children use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth or falsity of statements.

The National Policy on Education,1986 states that Mathematics should be visualized as the vehicle to train a child to think, reason, analyze and articulate logically. Apart from being a specific subject it should be treated as concomitant to any subject involving analysis and reason. The National Curriculum Framework for School Education (NCFSE, 2005) echoes such statements, but despite this Mathematics education has remained much the same, focused on narrow aims.

6. Children understand the basic structure of Mathematics: arithmetic, algebra, geometry and trigonometry. The basic content areas of school Mathematics, all offer a methodology for abstraction, structruration and generalization.
7. Teachers engage every child in class with the conviction that everyone can learn Mathematics.

5. MATHEMATICS ALL AROUND US AND ITS RELATION WITH OTHER SUBJECTS

Mathematics is all around us and we use Mathematics everyday but are seldom conscious of this. The patterns in flowers and leaves, the path taken by a buzzing fly, the shape of a matchbox or a building, cooking in the kitchen, playing in the field etc all require mathematical ideas and thinking.

'EVERYDAY MATHEMATICS'

In many poor urban households, children participate in economic activities. In a many social and geographical contexts, one finds children engaging with Mathematics outside school. Almost no school curriculum gives any place to such everyday 'street Mathematics'. At best there may be an attempt to add some contextual details to enhance the 'interest' of children. Thus the Mathematics that child learns to do inside and outside school remaining separate and disconnected. Of course, the larger issue here is of the relation between the school curriculum, and life outside school. Since Mathematics is an abstract branch of knowledge, one may think that there is little to be said about its connecting with culture and everyday life.

However, this is not true-

- Every individual needs to use Mathematics, for counting, ordering etc, and for transactions like commodity purchase, discount, interest, taxes, etc. Higher Mathematics helps you develop skill in technology.
- Mathematics has rich connections with art and architecture. It can be seen in Nature as well.
- Mathematics has rich connections with music, particularly the rhythms of classical music. It also shows itself in the rhythms of classical dance.
- Street vendors, carpenters, masons etc who have not got any school education do a lot of Mathematics at work. The folk algorithms that they develop for themselves require much mathematical thought and are not only restricted to the area of numbers but also require calculations and estimations of space. In India, the

traditions of *kolam* and *rangoli* have a lot to offer the student of Mathematics, apart from being beautiful to look at.

- Mathematics is an ever expanding subject, drawing for its growth on the sciences and the creativity of mathematicians and their exploration of open problems.

RELATION WITH OTHER SUBJECTS

Mathematics is embedded in other subjects and other subjects are also embedded in Mathematics. Children should learn Mathematics in concert with other subjects to see these connections.

- Mathematics is the gateway for understanding and learning Science. Neglect of Mathematics diminishes our appreciation and understanding of the world.
- Knowledge of Mathematics helps to understand and analyze the Physical Sciences. It also helps in deriving formulae and generalizing facts.
- Mathematics plays a role in understanding, deriving and expressing laws such as Newton's laws, gas laws, laws of electrical phenomenon and electronics, etc.
- All chemical combinations and their equations are governed by mathematical laws.
- Industrial extractions and preparation of different chemicals need analysis based on Mathematical calculation and estimations.
- Data handling methods and statistical analysis help in the understanding of concepts in geography and economics.
- Statistical analysis also helps in drawing conclusions in the biological sciences and medical research.
- Grammatical and linguistic base of all languages make use of Mathematics.
- Music is a hidden exercise in Arithmetic, of a mind that is not conscious of dealing with numbers.

6. SUBJECT SPECIFIC ACADEMIC STANDARDS – COMPETENCIES / OBJECTIVES – ELEMENTARY AND HIGH SCHOOL

The curriculum in Mathematics is influenced by many considerations: the nature of the subject itself, psychology and learning theory as well as the needs of contemporary society and culture.

ELEMENATRY SCHOOL

- Any curriculum for elementary Mathematics must incorporate the progression from the concrete to the abstract. Starting with concrete experiences helps the child understand the connections between the logical functioning of their everyday lives to that of mathematical thinking. At the same time, there is a need to help children handle abstraction.
- Children need to be given space for problem solving, especially problems which present real life situations. They need to be encouraged to come up with and use more than one way/method of doing things.
- Mathematics games, puzzles and stories involving numbers are useful to enable children to make these connections and build upon their everyday understanding. Games – not be confused with open ended play – provide non-didactic feedback to the child, with a minimum amount of teacher intervention.
- The Mathematics syllabus for the elementary classes has to revolve around understanding and using numbers and the system of numbers, understanding shapes and spatial relations, measurement, handling data etc. In this the identification of patterns is central as it helps children make the transition from arithmetic to algebra.

HIGH SCHOOL

- "Skills are taught, concepts are caught." (This has been said by many Mathematics educators, in particular by P K Srinivasan.)
- At this stage Mathematics comes to the student as an academic discipline. At the elementary stage, Mathematics education must be guided more by the logic of

child psychology than by the logic of Mathematics. But by the secondary stage, the student begins to perceive the logical structure of Mathematics. The notions of argumentation and proof become central.

- Geometry and trigonometry are wisely regarded as the area wherein students can learn to appreciate this structure best. Another area which can be used is Number Theory; e.g., tests of divisibility; properties of the sequence of positive integers.
- Algebra, introduced earlier, is developed at some length at this stage. Facility with algebraic manipulation is essential, not only for applications of Mathematics, but also internally, in the discipline itself. Proofs in trigonometry and geometry show the usefulness of algebraic machinery. It is important to ensure that students learn to visualize geometrically what they are able to accomplish algebraically.
- The student needs to integrate the many techniques of Mathematics into a problem solving ability. For e.g. this implies a need for posing problems to students which involve more than one content area: algebra and trigonometry, geometry and mensuration and so on. Trigonometry is used to prove results in Euclidean geometry; for example the theorem of Apollonius. (Note however that it is *not* possible to prove Pythagoras's theorem using trigonometry, as this involves circular reasoning. The student needs to be aware of such logical traps, which are quite common).
- A graded exposure to non-routine problems is essential, right from the early years. This exposure must be gradual. It is pointless either to pose problems that are beyond a child's ability, or so simple that they do not in any way challenge the child. Only a teacher who has worked with the children and knows them well would be able to decide the right level of problems to pose.
- Mathematics is used in the Physical and Social Sciences, and making the connections explicit can inspire students immensely.
- Mathematical modeling, data analysis and interpretation can consolidate a high level of mathematical literacy.
- An emphasis on experimentation and exploration is worthwhile. Mathematics laboratories are a recent phenomenon which will expand in the future. Activities in practical Mathematics help students greatly in visualization.

Concept Note on Academic Standards

What is Academic Standards : Academic standard specify what student should know and be able to do what is to be learned at certain point in time, and from a broad perspective, what performances will be accepted as evidence that the learning has occured.

To understand this firstly one has to know what is important to learn in mathematics for the student:-

1. First important thing is to understand the basic structure of mathematics with deep conceptual understanding

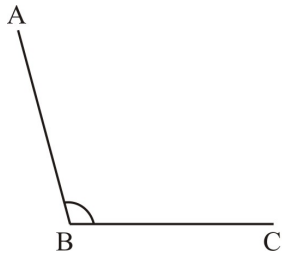
   in order to make sense of what they are doing, just knowing the procedure and memorizing of formulae is not enough ……**So estimation and question making** are the fundamental part which a child will be able to do to assert that she get some sense of what is taught to her

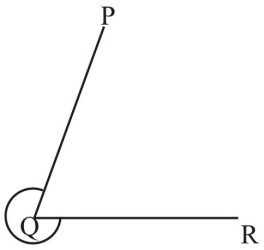
2. Second to master specific knowledge necessary for its application to real problems, for the study of related subject matter, and for continued study in mathematics.

To achieve the above goals as a main goals teacher has to emphasis in his teaching on guess work (estimation), encourage student to build on informal ideas in a gradual but structured manner so that they acquire the concept……., In this sense academic standard gives the guideline to teacher what and how teaching should occur in the classroom.

A example of this could be, Suppose after the completion of the topic "angle and its measurement" the most important question that every child should answer is that she could tell from the figure below , Which angle is smaller than other.(without measurement)

If you analysis this question , you come to a conclusion that without knowing what really a angle mean one could not give answer to that question. One can further change the same question and ask : now what is the bigger angle among these two?





Why are academic standards necessary?

Standard serve as rigorous goals for teaching and learning. Clear statements about what students must know and be able to do are essential to ensure that our schools offer students the opportunity to acquire the knowledge and skills necessary for success.

To get these goal, the ways (process) to approach must be cleared first. These ways are depends on different areas and content. Major we can categorized these in five broad ways.

Problem Solving

Mathematics is important because its concepts and procedures can be applied to the solution of problems of varying kinds and complexity. Solving problems challenges students to apply their conceptual understanding in a new or complex situation, to exercise their basic skills, and to see mathematics as a way of finding answers to many of the problems they encounter both within and outside the classroom. Students grow in their ability and persistence in problem solving through extensive classroom experience in posing, formulating, and solving problems at a variety of levels of difficulty and at every level in their mathematical development.

Reasoning

The ability to reason is such a vital part of mathematical behavior that it is safe to assert that mathematics cannot be done without it. At all levels, students should be able to provide a reason why they have chosen to apply a particular skill or concept, or why that skill works the way it does. Further, students should habitually check their results and conclusions for their reasonableness; that is, “does this make sense?” Proportional and spatial reasoning are specific kinds of reasoning that all students should have at their disposal. And, finally, it is important that all students should be able to apply the logical reasoning skills of induction and deduction to make, test, and evaluate mathematical conjectures, to justify steps in mathematical procedures, and determine whether conclusions are valid by analyzing an argument.

Communication

Whether working alone, or as part of a team, students must be able to communicate their thinking to others. Students must learn not only the signs, symbols, and specialized terms of mathematics, but also how to use this mathematical language in oral, symbolic, and written communication.

Connections

Mathematics should be viewed as a unified whole made up of connected, big ideas rather than as a disjointed collection of meaningless, abstract ideas and skills. Learning is easier when students see the connections between various concepts and procedures, and between the various branches of mathematics. Students should also be aware of the connections between, and applications of, mathematics and other disciplines, such as the sciences, art, music, business, medicine, and government.

Representation :

In representation we are looking for the competency where child read data, table, visualize and interpret it. We are also looking for the ability of making pictorial diagrams and associate the crude data or symbol to it.

7. CLASSROOM MANAGEMENT

Managing a Mathematics classroom requires a teacher to engage all children in the classroom. This is possible when a teacher understands that all children can learn Mathematics and that children can have different paces of learning.

Appreciating this, a teacher should construct different tasks for different groups of children. Collaborative learning among children would be a good option in many cases and in some others whole class teaching would be required. The choice of the learning task to be given and decision of having children work individually or in pairs or in small groups or to address them as a whole class are important decisions that a teacher takes in organizing and managing a class.

Managing a Mathematics classroom also requires acknowledging that various forms of social discrimination also work in the context of Mathematics education. Gendered attitudes consider 'Mathematics unimportant for specific categories of children' or 'girls incapable of learning Mathematics'. Similar beliefs exist for children belonging to certain castes. Both these need to be challenged in the classroom. Children with special needs, especially children with physical and mental challenges have as much right as every other child to learn Mathematics and their needs have to be addressed seriously. Thus, inclusion is a fundamental part of classroom management.

Decisions with respect to the teaching learning methods and materials that a teacher utilizes in the classroom are integral in classroom management decisions.

A TEACHING LEARNING STRATEGIES / METHODS – LEARNING ENVIRONMENT

Mathematics is a part of the daily life experiences for all of us. In playing cricket or any other sports children follow many rules and conditions when different situations arise and use them as a whole. They prove, disprove and argue giving reasons with established rules. In Andhra Pradesh girls create different patterns in front of their houses using lines, triangles and closed curves with different symmetries. Children use estimation, problem solving, approximation, reasoning skills in shopping. Making these experiences a part of classroom teaching lies at the core of 'mathematisation of the child's thinking'.

Also, it is often the case that people who are not formally educated or never enrolled in schools use many modes of mental Mathematics. For e.g. children of parents who work

as masons, plumbers, cobblers, tailors and artisans have a mathematical learning environment in their houses. A mason digs a well without knowing formal Mathematics. What may be called 'folk algorithms' exist for not only mentally performing number operations but also for measurements, estimation and understanding of shapes and aesthetics. If children are motivated to explore linkages between school and life situations, a natural mathematization process can take root in them.

The learning environment in the school must also give space to the whole range of mathematical processes- problem solving, use of heuristics, estimation and approximation, use of patterns, visualization, representation, reasoning and proof, making connections and mathematical communication.

The following activities may be used to enhance learning-

1. Problem posing and solving. Real life problems can also be analyzed in a group and more ideas incorporated as a result of group discussion.
2. Finding algorithms in Mathematics. Testing algorithms. Finding more than one way/method of doing things.
3. Making conjectures, building arguments, testing them, generalize them and verifying results.
4. Mathematical quizzes are a good medium for sparking interest in problem solving. These could be within a school or across schools in the form of a competition. The questions asked should be based upon the syllabus.
5. In the classroom students should be encouraged to present ideas, prepare talks and deliver them in front of other students and teachers. The topics may be from the regular Mathematics syllabus and also from the world of science and applications.
6. The formation of a Mathematics Club in a school can help create a stimulating mathematical environment in the school.
7. A Problem Corner can be started by the Mathematics Club, with suitable puzzles and mathematical problems at all levels.
8. Mathematical laboratories can have models of different kinds including geometrical shapes and solid objects like spheres, cubes, etc; charts of interesting curves; biographies of mathematicians; computers with Mathematics software; etc.

Posters, charts, equipment for explaining theorems or making measurements can be kept in this laboratory. These have to be supplementary and used as temporary scaffolds to help build abstract concepts.

9. Projects involving exploration. For e.g. collecting figures, photographs and models which have mathematical significance, from temples, mosques, churches, wall decorations etc, especially during festivals. Looking for patterns in nature.

10. Mathematics teaching can be made more interesting by telling students about the lives and works of some Mathematicians and relating the evolution of Mathematics to historical events. The story of development of various mathematical ideas and concepts can be very interesting and inspiring for children. Audio-visual displays can be prepared which will bring out the latent creativity of children, and the children can also do skits connected with historical themes.

TEACHING LEARNING RESOURCES - MATERIALS

Many resources and material are useful in teaching learning process such as textbooks, teaching learning material, math kit, information and communication technology (ICT), audio-visual aids and workshops.

- Textbook is the primary resource to the teacher and as well as the child. Teachers use it as a resource to plan lessons, understand concepts and evaluation procedures. Children use the textbook to engage with both concepts and processes.
- Workbooks for students and handbooks for teachers can be given for understanding and applying knowledge in many more contexts and various forms.
- Audio Visual aids may be used by the teachers in transformation of knowledge in a systematic presentation; for example Bulletin board, GeoBoard, Mathematics kit etc.

PRINCIPLES TO KEEP IN MIND FOR TEXTBOOK WRITING

- Textbooks should reflect the experiences of children. Situations that children encounter in real life should be used as far as possible for introducing concepts.

- Textbooks should be able to establish continuity with what children have previously learnt in the topic.
- Textbooks should use simple and unambiguous language. As far as possible they should act as self learning material for the student.
- Textbooks must be inviting in appearance and well structured.
- Each concept and process should be given with examples and exercises. Children should be given practice (in both concepts and process) in various contexts.
- Wherever possible solve problems using more than one method. Encourage children to do the same and also come up with their own ways of solving problems.
- All proofs need to be given in a non-didactic manner, allowing the student to see the flow of reason.
- Wherever possible more than one proof be given. Children be asked to analyse proofs and given incomplete proofs to complete.
- Children are generally introduced to proofs in geometry. But it is important for them to understand that a similar process is followed for numbers and thus be introduced to proofs in number theory too.
- Challenging questions to be provided at the end of each chapter.
- Add anecdotes about the history of Mathematics and achievements of mathematicians to make it interesting for children.
- If two new concepts are being introduced to the child in a class, then as far as possible a concept in which both these new concepts are being utilized together should not be introduced simultaneously. Example: if one learns Integers and the idea of a variable in introductory algebra for the first time in 7th standard then it is important that we do not pose problems that combine these two at the initial stage. There needs to be a gradual build up of concepts, and connections should be rationalized. Other such examples can be: Direct Proportion and Inverse Proportion; Algebra and Exponents. However, when these have been done for some time it is very important to give such mixed problems.

- The principle of appropriateness needs to be upheld. Concepts must not be introduced at a stage where they do not belong. Example: Limits and Proof by Mathematical Induction need not be introduced before the Senior Secondary stage (classes 11 / 12). The same is true for topics like Symmetric Polynomials and Cyclic Expressions in three variables, and the Double Angle formulas in Trigonometry. Similar decision need to be taken for other concepts in other classes too.

DIGITAL RESOURCES

The remarkable progress in information and communication technologies has made it possible for teachers to access quality digital resources and use them directly or indirectly in the classroom. Some unique features of these resources are the following:

1. Extensive availability of resources
2. Range of resources across topics and grades
3. Ease of sharing and recording of ideas and methodologies
4. Flexibility in usage – the resources can be used online or offline
5. Resources from other countries can be adapted for use in our schools with relative ease, because Mathematics is a universal subject
6. Availability of non-text resources like audio/video/applets
7. Availability of Open Source Software which can enhance a teacher's understanding of the subject: packages like GeoGebra and Logo. As they are freely available, they can be installed in every school/classroom equipped with a computer. Importantly, their usage encourages the formation of user communities whose members learn together and support one another; a practice that our teachers would do very well to adopt.
8. Low recurring cost
9. Increased penetration of IT in schools – both Urban and Rural

We must keep in mind that these resources can only be an aid and are not adequate by themselves. Teachers, also need to be prepared and made capable of using them.

8. TEACHER ROLE AND RESPONSIBILITIES

- Like other disciplines, Mathematics education also needs preparation with regard to understanding the nature of the subject, its content and pedagogic techniques of Mathematics.
- Mastery of the subject is an absolute essential for effective teaching; the level of teacher knowledge and understanding should be much higher than that he is expected to impart.
- The teacher should know not only Mathematics but also how Mathematics feeds into other subjects and vice-versa.
- He should acquire a good knowledge of educational psychology to be familiar with learning theory.
- Respect for a child's dignity, values and background are essential. A teacher should never make the mistake of humiliating a child in the name of feedback or learning.
- A teacher's personal characteristics are also important. These include general appearance, health, energy, enthusiasm, co-operation, sense of humor, sense of justice and willingness to speak the facts.
- Inadequate teacher preparation and support acts negatively on all of school Mathematics. Generally a teacher teaches how he learned from his teacher. Due to many pressures teachers adopt short cuts to achieve better marks and ranks. Learning achieved in this way is very short lived.
- Teachers should continue to want to be learners and also be open to unlearning. Some teachers assume that they know all the Mathematics and pedagogy they need to know.
- Each teacher should make definite provisions for his own professional growth and development. Institutional support is important in this context — support from the State educational institutions, and also from the authorities in the school to which the teacher belongs. This includes membership of professional bodies, and taking part in teacher development programmes which they conduct from time to time.

Every teacher must develop a network of co-teachers and teachers of higher level class teachers for clarification of doubts.

- Teacher should study appropriate and reputed mathematical journals and the latest books of professional development. Provisions should be made for making such materials more widely available, and at low cost.
- Teachers must make use technology like computers and the internet for the development of their own skills in teaching Mathematics in the classroom.

- As a part of his own preparation, teacher is required to observe classroom work of experienced teachers. He should also invite the same teachers to sit through his classes and ask for feedback.
- In-service training should be an essential part of continuing the training of teachers to a higher plane of efficiency and competence are commonly recognized. They should take relevant open university courses in mathematics and particularly mathematics teaching.
- Teachers have to maintain cordial and productive relationships with other teachers in the school to plan and execute the projects and for the efficient functioning of the school.
- Teachers should give students ample space in the classroom so that they can discuss, debate, voice their opinion and critique.
- Despite experience and prior knowledge a teacher must plan carefully and regularly for classroom teaching.
- Children with special needs especially those with mental and physical disabilities, have to be given equal importance as other children, with due care towards their disability. They need to be respected as human beings.
- Teacher must consciously undertake continuous evaluation of the student for better teaching and comprehensive assessment of the student.

International conference on Teacher Development and Management held in 2009 in Udaipur suggests that *'India needs to evolve an institution focused, comprehensive and well funded professional development plan for teachers. A plan that is life long in scope, based on demand, increasingly school based and innovative in terms of the models of*

training used. It must move beyond the programme and fund driven approach to in-service training'

The International seminar on Pre-service Elementary Teacher Education held in 2010 in New Delhi further suggests that-

- *'given the twin challenges of needing teachers in large numbers and improving the quality of teaching and learning in classrooms, develop multiple kinds and modalities of programmes for pre-service teacher preparation.*
- *Strengthen teacher education institutions by improving their infrastructure, resources and people capacity to respond to the challenges of pre-service teacher education.*
- *Pre-service and in-service training programmes should be conceptualized in continuation and as complementing each other. There should be linkages between the two that are drawn upon and explored; the understanding of teacher preparation as a one time pre-service certification course must end.*
- *End the isolation of teacher education institutions from larger academia and from other disciplines to keep it better informed of developments in research and pedagogic innovations and to introduce rigour and quality in their efforts.*
- *Strengthen linkages between various organizations and institutions involved in pre-service teacher preparation(such as DIETS and private colleges) and those in curriculum development and affiliation processes (such as SCERTs and NCTE)*

9. ASSESSMENT

Assessment is a major area of concern in the Indian education system. Whatever be the quality of teaching, textbooks, classroom, school leadership and so on, in the end it comes down to assessing the child; the school leaving certificate is what the child takes away with him or her from school, and for many children this represents the last point of contact with the subject. As the child receives a summative assessment (a pass/fail certificate) at this stage—a certificate which has a lasting consequence on the future of the child—it is vital that our assessment system be fair, honest and kind. At the present moment, unfortunately, it is very far from being this.

It is important that assessment should be based on more than just computational ability and committing to memory many formulas and solved problems; an assessment system must also test for conceptual understanding.

Furthermore, this cannot be left just to an examination taken at the end of the academic year. Examination oriented preparation lead to narrowness of vision and limited understanding. One's ability to ask questions, to pursue questions and to explore get blocked as a result. In the long run these negatively impact the child's self confidence, enjoyment and exploration. Assessment needs to be formative and not merely summative; it must be an integral part of the learning process i.e daily classroom activity from the start and should feed back into the teaching-learning process. Teachers should take pains to convey to the student that the purpose of assessment is not to label the child but to ascertain what are the areas of weakness and strength, in a nurturing and friendly atmosphere, and thus to use this feedback to grow.

The knowledge of the capability and the strength of the child as well as her way of approaching problems must be used by teacher to encourage her. Assessment needs to find out what the child knows and what she can learn rather than emphasise what she has not learnt and cannot learn at that time.

10. RECOMMENDATIONS

- Shifting of focus of Mathematics from "Narrow Goals" to "Higher Goals" (narrow aim only being: to turn out employable adults who contribute to social and economic development; higher goal: to develop all the inner resources of the growing child alongwith the narrow goal).
- Engaging every student with a sense of success.
- Provide conceptual challenges to the emerging mathematician.
- Change mode of assessment (mathematical abilities and the inner growth of the child should be the focus, rather than procedural knowledge or knowledge of facts and formulae). Board examinations to be restructured to reduce failures in Mathematics
- Enrichment of teachers with variety of ways-
    - Regular in-service training

- Network system to be developed among Mathematics teachers at all levels, from classroom teachers to professors of university (making full use of emerging web based resources)
- Teachers' advanced professional skills to be assessed periodically, and to provide necessary inputs that will enable this
- Mathematics content: journals and books
- Creation of math clubs and math laboratory
- Mathematics software, and instructional workshops that will encourage usage of such software
- Publication of a newsletter for Math teachers

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Constitution of India preamble

WE, THE PEOPLE OF INDIA, ***having solemnly resolved to constitute India into a***

SOVEREIGN, SOCIALIST, SECULAR, DEMOCRATIC, REPUBLIC ***and to secure to all its citizens***

JUSTICE, *social economic and political*

LIBERTY *of thought, expression, belief, faith and worship*

EQUALITY *of status and opportunity and to promote among them all*

FRATERNITY *assuring the dignity of the individual and the unity and integrity of the nation.*

IN OUR CONSTITUENT ASSEMBLY this twenty –**sixth day of November, 1949**, do HEREBY **ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.**